Brushed DC Motor Theory and Operation

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Abstract

This study looked into the effect of changing the material within the rotor of an electric motor. In a brushed DC motor, the coils on the rotor utilize the motor’s magnetic field to create torque on the axle. Within the rotor’s coil, we inserted different materials and observed the effect on the motor’s performance. To do this, we built a brushed DC motor with 6 magnets. The rotor included a cavity within the coils for inserting various materials. We planned to experiment by inserting 4 different materials into the cavity. The characteristics that would have varied were conductivity and reaction to an applied magnetic field. Unfortunately, the mechanical loss in our motor was too great to ever actually run this as a functional motor and compare the efficiency and torque with different materials. As an alternative, we ran it as a generator (turning the shaft and measuring the voltage) and captured the peak voltage output by the motor. This peak voltage enabled us to determine how the motor would respond at peak speed, as a motor and a generator are the same device, it’s just a matter of whether it’s driven mechanically to produce an electrical output or vice-versa. As expected, the non-conductive ferromagnetic material produced the largest electrical output.
1 Introduction

Electric motors play essential parts in many aspects of our life. Each application has varied requirements, though; for example, some motors need to be cheap, light, and draw minimal current while others have to be small and still have significant torque. One way to fulfill some of these use requirements is to alter the materials inside the motor, thereby altering its behavior. By choosing a ferromagnetic material, the motor could have a higher torque output, but may be more expensive to produce. In this study, we investigated four different types of materials that can be used to construct an electric motor. While we were unable to complete the original experiment, we were able to experiment with the materials inside the motors by using the motor as a generator. First, we will explain the calculations we made when designing our motor. Then, we will explain the experiment we planned and the motor we designed. Next, we’ll discuss our simulation that characterizes the internal magnetic fields of our motor and allows us to visualize them. Finally, we’ll explain why our experiment did not work as planned and the new experiment we used in its place.
2 Theory

Brushed DC motors, despite their mechanical complexity, are governed by only a few forces. First, a look at an incredibly simplified "motor", a square loop in a magnetic field, as shown in Figure 1.

![Simple Motor Diagram](image)

**DC motor: principle and simplified equations of motion**

\[ T = 2Fr = 2(iBN)l \]  
\[ v_e = 2VBNl = 2(\omega r)BNl \]  
\[ \text{or} \]
\[ T = K_mi \]
\[ v_e = K_\omega \]

- \( K_m = 2BNlr \) torque constant
- \( K_\omega = 2BNlr \) back-emf constant

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2.1 Defining Torque

The loop will feel a force exerted on it, and if allowed to rotate, this force will become a torque. From this setup the torque and angular velocity can readily be determined from fundamental laws. The torque can be defined as

\[ \tau = 2F \cdot r \]  

(1)

Each side of the loop will experience a force, but in opposite directions. Since the loop is allowed to rotate, however, the two forces will sum and rotate the system forward. Cross this force with the radius, to get the torque. That was just basic mechanics, the question that remains is how to determine the force that the loop experiences. This is where the Lorentz Force law comes...
into play. A fundamental law of electricity and magnetism, this law states

\[ F = i \times B \times N \times l \]  

(2)
in the direction of increasing theta.

\( i = \) Current (A)
\( N = \) number of windings
\( l = \) length of loop perpendicular to the B field

Now having defined the torque in terms of known values, it is also possible to characterize the torque constant of the motor. This value is different for every motor, dependent on a number of variables ranging from the friction in the bearing to the strength of the magnets. Since torque is linearly dependent to the input current, this constant is defined as

\[ T = K_m \times i \]  

(3)

Solving for \( K_m \) by substituting \( T \) with equation X1,

\[ K_m = 2 \times B \times N \times l \times r \]  

(4)

This definition will be used in the transfer function to calculate the response of this particular system.

2.2 Defining Back EMF

Another important aspect of the motor is calculating the back EMF, or the voltage generated as a result of the coils rotating within a magnetic field. Knowing the relationship between back EMF and angular velocity is also a method of calculating the motor speed at a given point in time without the need for an external encoder, which is particularly useful for a home brew motor like this one.

Using Faraday’s Law, the voltage generated by the coil rotating in the magnetic field is defined as

\[ V_c = 2 \times \omega \times r \times B \times N \times l \]  

(5)

\( V_c = \) The velocity of the coils (\( \omega \times r \))
\( B = \) magnetic field
\( N = \) number of windings
\( l = \) length of loop perpendicular to the field
Again, note that $V_e$ has only one time variant parameter, $\omega$, which then allows for the creation of a linear relationship between $\omega$ and $V_e$

$$V_e = K_e * \omega$$  \hspace{1cm} (6)

$$K_e = 2 * B * N * l * r$$  \hspace{1cm} (7)

Using this constant, it is possible to look at the voltage and calculate the expected angular velocity, or if the angular velocity is known, the expected voltage can be determined.

Additionally, it is important to note that for any motor, the torque constant, $K_m$ and the back emf constant, $K_e$ must be the same.

$$Power_{in} = Power_{out}$$

$$i * v_e = T * \omega$$

$$K_e * i * \omega = K_m * i * \omega$$

$$K_e = K_m$$

### 2.3 Deriving the Transfer Function

The transfer function of a DC motor begins with two fundamental relationships. Kirchoff’s current law (KCL) and Kirchoff’s voltage law (KVL).

Kirchoff’s current law states that at any node in the circuit, current is conserved, and Kirchoff’s voltage law states that the sum of voltage around a loop is 0.

Approximating the motor as an inductor and resistor in series utilizes these two laws.

There are 4 constitutive relations that are important here:

- Voltage across the Inductor

$$\frac{V_L}{L} = \frac{di_L}{dt}$$ \hspace{1cm} (8)

- Current across Resistor

$$\frac{V_R}{R} = i_R$$ \hspace{1cm} (9)

- Torque on Rotating Mass

$$\frac{\tau}{dt} = \frac{\tau_M - \tau_B}{I}$$ \hspace{1cm} (10)
Torque at Bearing

\[ \tau_B = K_B \omega \]  \hspace{1cm} (11)

Using the KCL and KVL relations, a number auxiliary equations can be written.
As a result of KVL:

\[ V_L + V_R + V_M = V_{in} \]  \hspace{1cm} (12)

KCL states that:

\[ i_L = i_r = i_m \]  \hspace{1cm} (13)

Then the motor relationships from before become:

\[ \tau_m = K_m \cdot i_m \]  \hspace{1cm} (14)

There are two unknowns in the system, current of the inductor, \( i_L \), and the angular velocity, \( \omega \). The energy stored in the system is dependent on these two parameters.
Now to solve for these two unknowns, several of the relationships defined earlier will be combined. Note that the current is assumed to be the same at every point.

\[ \frac{di_L}{dt} = \frac{1}{L} (V_{in} - i_L \cdot R - K_m \cdot \omega) \]  \hspace{1cm} (15)

\[ \frac{di_L}{dt} = \frac{1}{I} (K_m \cdot i_L - K_B \cdot \omega) \]  \hspace{1cm} (16)

Now using these relationships, the system will be described using state space form, setting up the Laplace transform.

\[ \frac{d}{dt} \begin{bmatrix} i_L \\ \omega \end{bmatrix} = \begin{bmatrix} -R \\ \frac{K}{L} \end{bmatrix} \cdot \begin{bmatrix} i_L \\ \omega \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \cdot V_{in} \]
\[
y = [0 \ 1] \times \begin{bmatrix}
 i_L \\
 \omega
\end{bmatrix} + 0 \cdot V_{in}
\]

2.4 Deriving the Transfer Function

Assuming all initial conditions equal zero and that \( D = 0 \).

\[
s \cdot X(s) = A \cdot X(s) + B \cdot U(s)
\]

\[
Y(s) = C \cdot X(s) + D \cdot U(s)
\]

\[
X(s) = (s \cdot I - A)^{-1} \cdot B \cdot U(s)
\]

\[
Y(s) = C \cdot (s \cdot I - A)^{-1} \cdot B \cdot U(s) + D \cdot U(s)
\]

The transfer function is defined as \( \frac{Y(s)}{U(s)} \), therefore, the transfer function evaluates to

\[
\frac{Y(s)}{U(s)} = C \cdot (s \cdot I - A)^{-1} \cdot B
\]

Substituting in the matrices, and the values will produce

\[
G(s) = \frac{-K_m \cdot m}{l \cdot s^2 - \left( -B \cdot \frac{R}{L} + \frac{B}{l} \cdot \frac{R}{L} \right) - m \cdot l}
\]

3 Experiment

To truly understand the effects of material choice in electric motors, we had to design our own electric motor. It was very clear from the beginning that a brushed DC motor was the best fit for our needs. It was simple to comprehend, model, and fabricate. The first major design decision was to build a motor that would facilitate our experiment as effectively as possible. Using four rotors made out of four different materials would allow us to get very complete data, but would have been very time and material intensive. Instead, we designed one rotor out of plastic with a large cavity within both of the coils, as can be seen in Figure 2. This allowed us to insert different materials into the rotor instead of switching out the rotor entirely.

The parts were fabricated entirely at Olin – the only materials we didn’t make were the bearing and the magnet segments. The rotor was made in the 3d printer due to having a shape that is very difficult to machine conventionally.

On the rotor, we coiled wire on each arm 40 times. Our commutator, a normal split-ring commutator with six sections, was made on the out of conductive tape, and mounted on our axle, which was turned down on the lathe. The brushes
are simply multi-strand wire threaded through the base, fanned out, and held against the commutator.

Figure 2: Cross-sectional view of rotor showing cavities.

For our shell, we used 2” diameter PVC pipe which we bored out slightly, to match the diameter of the magnet. In the housing, we epoxied 6 magnets an equal radial distance apart to create the magnetic field shown in Figure 3. Finally, for the endplates, we milled acrylic bases which held the shell and commutator in place. The commutator was held with a ball bearing so it could spin freely.

When choosing materials for our inserts, one of the characteristics we took into account was the material’s tendency towards forming eddy currents. Eddy currents are currents that occur within a material opposing the change in magnetic field. In nonconductive materials, eddy currents can be ignored, because the material’s resistance keeps them from having much of an effect. In conductive materials, these currents dampen the magnetic fields that make the motor run. In real electric motors, there are several strategies for increasing internal resistance and minimizing eddy currents in conductive materials. Some motors use electrical steel, which contain silicon, a semiconductor that substantially increases resistance[4]. Other motors separate their conductive materials with layers of nonconductive materials[4]. Less commonly, some electric motors use powdered material in nonconductive resin[4]. The isolation increases the resistivity. For our experimental materials, we chose some conductive and some nonconductive materials.

The second characteristic we considered when choosing materials was whether
the material was ferromagnetic. Ferromagnetic materials create a relatively strong magnetic field in the presence of an applied magnetic field. This increased magnetic field augments the force pushing on each side of the rotor allowing our motor to output more torque.

With these two characteristics in mind, we chose four materials to test our motor with, hitting the major combinations of materials that are ferromagnetic and those that induce eddy currents. We chose to use plastic (which is not magnetically conductive or electrically conductive), aluminum (electrically conductive but not magnetically conductive), steel (both electrically and magnetically conductive), and ferroceramic magnets (magnetically conductive but not electrically conductive). This enabled us to test the effect of both eddy currents and ferromagnetic materials on the motor’s performance, by creating 4 tests which are different combinations of the two factors that influence our motor’s performance. The difference between conductive and nonconductive materials is a massive one; and so the multiple-order-of-magnitude difference between materials which are conductive and which are not greatly overwhelms the small difference between different conductive and nonconductive materials.
4 Simulation

In order to simulate the motor we had attempted to construct, we decided to use the physics modeling program COMSOL. This software allows us to both create a 3D representation of our motor and model the electromagnetics happening within. While COMSOL does allow importing of existing CAD models, this functionality is incredibly error prone and problematic, so we decided to forgo using our existing computer model and re-create the geometry using COMSOL’s tools to do so. Since these tools are primitive in comparison to many other CAD programs, we had to make many simplifications to the design, however, these assumptions will not affect the accuracy of the simulation in regards to characterizing the physics of the motor. The first simplification we had to make was to model our magnets as rectangles, omitting the curved shape that our magnets actually possess. Since we are only concerned with how the magnetic fields behave, the geometry change is negligible. The magnet still behaves as a dipole, with opposing poles on opposite faces of the magnet. Since COMSOL does not offer a convenient way to define a magnetic field, we had to first derive the equivalent surface current of our magnets.

To find the surface current, we approximated our magnet as a solenoid. Using the the known field at the edge of the magnet, \(310T\), we utilized the equation for the magnetic field of an inductor \(B = \mu NI\). \(NI\) is essentially our surface current, which we solved for and plugged into COMSOL. As can be seen in Figure 4, the resultant field for a single magnet in this configuration is approximately \(.36T\), which is well within an order of magnitude of what we measured. The small discrepancy could even be accounted for by the limit to how close we could take a measurement of the actual magnets.

After getting one magnet working correctly, we added the remaining magnets to correctly model the motor we had built. As expected, the strength of the magnetic fields drops off significantly toward the center of the arrangement where all magnets are equidistant. This effect can be seen in Figure 5.

5 Analysis

Unfortunately, our motor was not functional. This is due in large part to the amount of friction in our commutator. This is the result of a design flaw where we soldered the connecting wires on top of our commutator plates, and so we did not have a cylindrical surface for our brushes to run over. If we had this to do over, we would make a change as shown below in Figure 6 to fix this problem.
In this design, the commutator plates have tabs which protrude upwards, so that wires can be soldered on without damaging the cylindrical surface that the split-ring commutator have to connect with the brushes. These tabs will also allow for better indexing of our commutator to our axle. The new axle design could be made utilizing CNC technologies to guarantee high-tolerance alignment of the commutator plates, making sure we have polarity switches exactly where we want them.

Because our motor is relatively small, hitting the tolerances necessary for things to work smoothly with the way our commutator was designed was very hard. To make this work better, we either would have needed a bigger motor or a self-jigging way of manufacturing to limit human error in assembly.

Another issue is that, because our rotor is made of plastic, which is magnetically nonconductive; the gap between the permanent magnet and the electromagnet on the rotor is large – the circular profile of our rotor doesn’t actually gain us anything given the material that our rotor is made out of, which is FDM plastic. Were it made out of a ferromagnetic material, the air gap for flux to cross would be much smaller, and our motor would be much more efficient.

Even without a functional motor, we were still able to plan and perform a dif-
different experiment. Instead of using our motor as motor by giving it an electrical input and receiving a mechanical output, we used our motor as generator by turning the axle by hand. We could then measure the corresponding signal. We were able to insert our materials into the motor as planned and see what they

Figure 5: Magnetic fields inside motor housing.

Figure 6: Revised design of the commutator.
affected this corresponding signal.
We were only really able to record data for the case where the rotor was stuffed
with ferroceramic material, the others did not generate enough of a voltage peak
to register well on the oscilloscope. When we stuffed the cores with ferroceramic
materials, we were able to register some measurable voltage when turning it – re-
peatedly peaking at 420 millivolts, an example of which can be seen in Figure 7.
This peak voltage of 420 mV can be input into our transfer function to calculate the speed that we were turning the motor.

\[
\omega = \frac{v}{2 * B * N * l * r}
\]

\[
\omega = \frac{0.420}{2 * 0.31 * 80 * 0.011938 * 0.0225} = 31.5 \text{ rad/sec}
\]

A quick turn of the wrist could definitely result in 31.5 radians per second, giving validity to our calculations based on real world results.

6 Conclusion

All in all, the project was a relative success. Although the motor did not run smoothly, there were a few choice moments where the rotor moved on its own for a few steps, but never was a full cycle completed. Our team spent a lot of time researching motor construction, as well as studying the theory behind its operation. Everything from attempting to emulate professional commutator designs, to deriving the transfer function from fundamental laws, this project has really increased our collective knowledge of brushed DC motors. Additionally, instead running the motor, we used it as a generator, generating a steady 420 mV peak, indicating that the system was built correctly, but mechanical resistance in the bearing and commutator were too great to overcome. In the future, it would be beneficial to invest in smoother bearings, rethink the commutator design, and build the rotor ends with ferrous materials to better focus the flux.

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8 References

9/17/07.


9 Appendix

Figure 8: Exploded View of the Motor
Figure 9: Multiple views of the bottom of the housing.
Figure 10: Multiple views of the top of the housing.
Figure 11: Multiple views of the motor axle and commutator.
Figure 12: Multiple views of the motor housing.